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An analytical study of the dynamic characteristics for resonant vibratory micro electro-mechanical gyroscopes system

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Abstract— in this paper, the dynamic characteristics and nonlinear behavior for resonant vibratory micro-electro-mechanical gyroscopes system (MEMS) are studied. The MEMS gyroscopes system described by nonlinear differential equations including linear terms with external excitation force. The multiple scale perturbation technique (MSPT) is applied to derive the approximate mathematical solutions of the governing equations up to the second order approximation. The different worst resonance cases is reported and studied numerically using Runge-Kutta of fourth order. We applied both frequency response equations and phase-plane technique to analyze the stability of the steady state solution of vibrating system. Numerical simulations are presented to verify the effectiveness of the different parameters on the micro electro-mechanical gyroscopes system using MATLAB and MAPLE programs. Results are compared to previously published work.

Index Terms— Micro-Electro-Mechanical System (MEMS) gyroscopes, Stability, Resonance, Vbrations.

1 Introduction

Micro electro-mechanical systems (MEMS) are increasingly being used in measurement and control problems due to their small size, low cost, and low power consumption. The vibrating gyroscope is a MEMS device that will have a significant impact on stability control systems in the transportation industry. The vibrating gyroscope is one of the MEMS devices that commonly used for measuring angular velocity. MEMS gyroscopes are widely applied in the area of aviation, navigation, automotive, biomedicine, military affairs, and consumer electronics. The performance of the MEMS gyroscopes is deteriorated by the effects of time varying parameters, environment variations, quadrature errors, and external disturbances. Therefore, advanced control such as adaptive control, sliding mode control, and intelligent control are necessary to be used to control the MEMS gyroscope and improve its performance and stability. Engineering researchers have paid much attention to studying the non-linear dynamics, bifurcations, chaos and stability of the vibrate MEMS gyroscopes.

There are two main ways to control this vibration the active and passive control. Egretzberger et al. [1] proposed the design of open and closed-loop controllers for vibratory MEMS gyroscopes models. Braghin et al. [2] investigated that the resonance peak of the structure bends towards the higher frequencies when the nonlinear hardening characteristic of the supporting beams becomes visible. This property is useful to easily sense and drive resonances thus increasing the sensibility of the MEMS gyroscope. A parametrically amplified MEMS rate gyroscope were studied by Hu et al. [3] and the response of the controller was investigated experimentally and demonstrated stable amplification in the presence of step changes in the input. Yoon et al. [4] presented that the vibration effects on MEMS degenerate gyroscopes by vibratory ring gyroscopes. Riaz et al. [5] investigated that a 3-DOF nonresonant micro gyroscope design concept with structurally decoupled 2-DOF drive-mode and 1-DOF sense-mode oscillator. A dynamic amplification of 3 times at first resonant frequency was achieved by the passive mass and 9 times at second resonant frequency in comparison with the active mass. Fei and Xin [6] proposed an adaptive fuzzy sliding mode control (AFSMC) for Micro-Electro-Mechanical Systems (MEMS) triaxial gyroscope with angular velocity. Sung et al. [7–9] provided resonance in both modes through phase-locked loop (PLL) control of MEMS vibratory gyroscope. Asokanthan and Wang [10] investigated the nonlinear instabilities in a single-axis vibrating MEMS gyroscope and studied the bifurcation behavior associated with the steady state. The method of averaging employed to solve coupled differential equations. Pak-

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niyat et al. [11] studied the stability and the effects of different parameters of a parametrically resonated MEMS gyroscope with harmonic excitation. Suketu Naik et al. [12] they showed that the multiple bifurcations depending on the interaction between damping constant, excitation amplitude and excitation frequency for the coupled of Micro electromechanical Systems. The coupled system showed phase-locked behavior and full entrainment to the excitation force at a higher value of the excitation amplitude. They reported that the coupled system can be used as a sensor depending on the target application. Mrigank Sharma et al. [13] they studied the sensing mode of a MEMS gyroscope with parametric amplification and damping. They amplified the mechanical oscillations or reduced the unwanted oscillations by controlling the phase difference between the excitation and the parametric actuation. Experiments confirmed that parametric modulation through electromechanical coupling leads to both an increase in spectral selectivity and a reduction of the equivalent input noise angular rate for a parametric gain. Juntao Fei et al. [14] developed a fuzzy logic-based adaptive sliding mode controller and adaptive fuzzy sliding mode controller with bound estimation to control the trajectory of an angular velocity sensor and relax the requirement for the bound value in the sliding control. The stability of the closed-loop system can be guaranteed with the proposed adaptive fuzzy control strategy with bound estimation. Simulations are implemented to verify the effectiveness of the proposed adaptive fuzzy control and demonstrate that the proposed adaptive fuzzy control system with bound estimation yields superior control performance. Kamel and Hamed [15] studied the nonlinear behavior of an inclined cable subjected to harmonic excitation near the simultaneous primary and 1:1 internal resonance using multiple scale method. Hamed et al. [16-18] showed how effective is the passive vibration control reduction at resonance under multi-external or both multi-external and multi-parametric and both multi-external and tuned excitation forces. They reported that the advantages of using multi-tools are to machine different materials and different shapes at the same time. This leads to saving the time and higher machining efficiency. Hamed et al. [19] presented the behavior of the nonlinear string beam coupled system subjected to external, parametric and tuned excitations for case 1:1 internal resonance. The stability of the system studied using frequency response equations and phase-plane method. It is found from numerical simulations that there are obvious jumping phenomena in the frequency response curves. Sayed and Hamed [20] studied the numerical response and stability analyses of a two-degree-of-freedom system under harmonic and parametric excitation forces. They obtained the approximate solutions up to and including the second-order approximations using the method of multiple scale perturbation technique. Sayed and Kamel [21, 22] investigated the influence of different controllers on the vibration control system. They reported that, the saturation of non-linear vibration absorbers is used to reduce and control the movement due to rotor blade flapping. Amer and El-Sayed [23] investigated the nonlinear dynamics of a two-degree-of freedom vibration system with absorber when subjected to multi external forces at primary and internal resonance with ratio 1:3. They reported that the steady-state amplitude of the main system is reduced to 2.5% of its maximum value. Kamel, Eissa and EL-Sayed et al. [24-28] obtained the results of effectiveness of the transverse or the longitudinal or both non-linear controllers on vibration of the ship roll system with multi-external or multi-parametric or both multi-parametric and multi external excitation. They introduced a study on the stability and effect of parameters on the vibration of spring pendulum system and controllers. Sayed and Mousa [29] investigated the influence of the quadratic and cubic terms on non-linear dynamic characteristics of the angle-ply composite laminated rectangular plate with parametric and external excitations. The method of multiple time scale perturbation is applied to solve the non-linear differential equations describing the system up to and including the second-order approximation. Two cases of the sub-harmonic resonances cases ($\Omega_2 \cong 2\omega_1$ and $\Omega_2 \cong 2\omega_2$) in the presence of 1:2 internal resonance $\omega_2 \cong 2\omega_1$ are considered. The stability of the system is investigated using both frequency response equations and phase-plane method. It is quite clear that some of the simultaneous resonance cases are undesirable in the design of such system as they represent some of the worst behavior of the system. Such cases should be avoided as working conditions for the system. Sayed and Mousa [30] studied an analytical investigation of the nonlinear vibration of a symmetric cross-ply composite laminated piezoelectric rectangular plate under parametric and external excitations. Their study focused on the case of 1:1:3 primary resonances and internal resonance, and they verified the analytical results calculated by the method of multiple time scale by comparing them with the numerical results of the modal equations. The obtained results were verified by comparing the results of the finite difference method (FDM) and Runge-Kutta (RKM) method. Kamel et al. [31] studied a model subject to multi-external excitation forces. The model is represented by two-degree-of-freedom system consisting of the main system and absorber simulating ultrasonic machining. They used the passive vibration controller to suppress the vibration behavior of the system. Mousa et al. [32] investigate the stability of a simply supported laminated composite piezoelectric rectangular plate under combined excitations. The analytical results are verified by comparing them with those of numerical integration of the modal equations. The influence of different parameters on the dynamic behavior of the composite laminated piezoelectric rectangular plate is studied. Variation of the some parameters leads to multivalued amplitudes and hence to jump phenomena.

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2. Mathematical Modeling

The non-dimensional equation of motion described the MEMS gyroscope system can be given by [14]

$$\ddot{x} + \alpha_1 \dot{x} + \alpha_2 \dot{y} + \omega_1^2 x + \beta y = f_1 \sin \Omega_1 t + \gamma \dot{y} \quad (1)$$

$$\ddot{y} + \alpha_2 \dot{x} + \alpha_3 \dot{y} + \beta x + \omega_2^2 y = f_2 \sin \Omega_2 t - \gamma \dot{x} \quad (2)$$

where x and y are the co-ordinates of the proof mass with respect to the gyro frame in a Cartesian co-ordinate system; α_1 and α_3 are damping; α_2, β called quadrature errors, are coupled damping and spring terms, respectively, mainly due to the asymmetries in suspension structure and misalignment of sensors and actuators. The coupled spring and damping terms are unknown, but can be assumed to be small. The nomial values of the x and y axes spring and damping terms are known, but there are small unknown variations; ω_1 and ω_2 the linear natural frequencies, and Ω_1, Ω_2 the excitations frequencies, f_1, f_2 are the amplitudes of external excitation forces, $\gamma \dot{y}$ and $\gamma \dot{x}$ are the Coriolis forces.

The linear viscous damping forces, quadrature errors and exciting forces are assumed to be

$$\alpha_n = \varepsilon \hat{\alpha}_n,\ \beta = \varepsilon \hat{\beta},\ f_1 = \varepsilon \hat{f}_1,\ f_2 = \varepsilon \hat{f}_2 \quad n = 1,2,3 \quad (3)$$

where ε is a small perturbation parameter and $0 < \varepsilon << 1$. The behavior of such a system can be very complex, especially when the natural frequencies and the forcing frequency satisfy certain internal and external resonance conditions. The primary resonances case $\Omega_1 \cong \omega_1$ in the presence of 1:1 internal resonance $\omega_2 \cong \omega_1$ is considered. To describe how close the frequencies are to the resonance conditions we introduce detuning parameters:

$$\Omega_1 = \omega_1 + \sigma_1 = \omega_1 + \varepsilon \hat{\sigma}_1, \quad \omega_2 = \omega_1 + \sigma_2 = \omega_1 + \varepsilon \hat{\sigma}_2 \quad (4)$$

where σ_1 and σ_2 are called the external and internal detuning parameters, respectively. The parameters $\hat{\alpha}_n, \hat{\beta}, \hat{f}_1, \hat{f}_2$ and the detuning parameters $\hat{\sigma}_1$ and $\hat{\sigma}_2$ are of order 1. We determine a first-order approximation for the system response by the method of multiple scales [33-34], which is a powerful tool in determining periodic solutions of small amplitude.

Equation (1)-(2) rewritten in the form:

$$\ddot{x} + \varepsilon \hat{\alpha}_1 \dot{x} + \varepsilon \hat{\alpha}_2 \dot{y} + \omega_1^2 x + \varepsilon \hat{\beta} y = \varepsilon \hat{f}_1 \sin \Omega_1 t + \varepsilon \hat{\gamma} \dot{y} \quad (5)$$

$$\ddot{y} + \varepsilon \hat{\alpha}_2 \dot{x} + \varepsilon \hat{\alpha}_3 \dot{y} + \varepsilon \hat{\beta} x + \omega_2^2 y = \varepsilon \hat{f}_2 \sin \Omega_2 t - \varepsilon \hat{\gamma} \dot{x} \quad (6)$$

The approximate solution of Eqs. (5)-(6) can be obtained by using the method of multiple scales [33-34]. Let

$$x(t;\varepsilon) = x_0(T_0, T_1) + \varepsilon x_1(T_0, T_1) \quad (7)$$

$$y(t;\varepsilon) = y_0(T_0, T_1) + \varepsilon y_1(T_0, T_1) \quad (8)$$

where, $T_n = \varepsilon^n t$ (n = 0, 1) are the fast and slow time scales respectively. In terms of T_0 and T_1, the time derivatives transform according to

$$\frac{d}{dt} \equiv D_0 + \varepsilon D_1, \quad \frac{d^2}{dt^2} \equiv D_0^2 + 2\varepsilon D_0 D_1 \quad (9)$$

where $D_n = \partial / \partial T_n$. Substituting Eqs. (7)-(9) into Eqs. (5)-(6) and equating coefficients of similar powers of ε, one obtains:

Order (ε^0):

$$(D_0^2 + \omega_1^2) x_0 = 0 \quad (10)$$

$$(D_0^2 + \omega_2^2) y_0 = 0 \quad (11)$$

Order (ε^1):

$$(D_0^2 + \omega_1^2) x_1 = -2 D_0 D_1 x_{10} - \hat{\alpha}_1 D_0 x_0 - \hat{\alpha}_2 D_0 y_0 - \hat{\beta} y_0 + \hat{\gamma} D_0 y_0 + \hat{f}_1 \sin \Omega_1 t \quad (12)$$

$$(D_0^2 + \omega_2^2) y_1 = -2 D_0 D_1 x_{20} - \hat{\alpha}_2 D_0 x_0 - \hat{\alpha}_3 D_0 y_0 - \hat{\beta} x_0 - \hat{\gamma} D_0 x_0 + \hat{f}_2 \sin \Omega_2 t \quad (13)$$

The solution of Eqs. (10)-(11) can be expressed in the complex form:

$$x_0 = A \exp(i \omega_1 T_0) + cc \quad (14)$$

$$y_0 = B \exp(i \omega_2 T_0) + cc \quad (15)$$

where A and B are a complex function in T_1 and cc stands for the complex conjugate of the preceding terms. Substituting Eqs. (14)-(15) into Eqs. (12)-(13) and using the resonance conditions Eq. (4) lead to secular terms. Eliminating these secular

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terms leads to the solvability conditions for the first-order expansion:

$$2i\,\omega_1 D_1 A = -i\,\hat{\alpha}_1\omega_1 A - (i\,\hat{\alpha}_2\omega_2 + \hat{\beta} - i\,\hat{\gamma}\omega_2)B\,\exp(i\,\sigma_2 T_1) + \frac{\hat{f}_1}{2i}\exp(i\,\sigma_1 T_1) \qquad (16)$$

$$2i\,\omega_2 D_1 B = -i\,\hat{\alpha}_3\omega_2 B - (i\,\hat{\alpha}_2\omega_1 + \hat{\beta} + i\,\hat{\gamma}\omega_1)A\,\exp(-i\,\sigma_2 T_1) \qquad (17)$$

For the first order approximations, we get:

$$2i\,\omega_1\frac{dA}{d\,t} = \varepsilon\, 2i\,\omega_1 D_1 A\,,\quad 2i\,\omega_2\frac{d\,B}{d\,t} = \varepsilon\, 2i\,\omega_2 D_1 B \qquad (18)$$

To analyze the solutions of Eqs. (16)-(17), we express A, B in the polar form

$$A = \frac{1}{2}a\,e^{i\varphi_1}\ ,\quad B = \frac{1}{2}b\,e^{i\varphi_2}\ , \qquad (19)$$

where a,b and φ_1,φ_2 are the steady state amplitudes and phases of the motion respectively. Substituting Eqs. (16)-(17) and (19) into Eq. (18) and equating the real and imaginary parts we obtain the following equations describing the modulation of the amplitudes and phases of the response:

$$\dot{a} = -\frac{\alpha_1}{2}a - \frac{\beta}{2\omega_1}b\sin\theta_1 - \frac{(\alpha_2-\gamma)\omega_2}{2\omega_1}b\cos\theta_1 - \frac{f_1}{2\omega_1}\cos\theta_2 \qquad (20)$$

$$a\dot{\varphi}_1 = \frac{\beta}{2\omega_1}b\cos\theta_1 - \frac{(\alpha_2-\gamma)\omega_2}{2\omega_1}b\sin\theta_1 - \frac{f_1}{2\omega_1}\sin\theta_2 \qquad (21)$$

$$\dot{b} = -\frac{\alpha_3}{2}b + \frac{\beta}{2\omega_2}a\sin\theta_1 - \frac{(\alpha_2+\gamma)\omega_1}{2\omega_2}a\cos\theta_1 \qquad (22)$$

$$b\dot{\varphi}_2 = \frac{\beta}{2\omega_2}a\cos\theta_1 + \frac{(\alpha_2+\gamma)\omega_1}{2\omega_2}a\sin\theta_1 \qquad (23)$$

where

$$\theta_1 = \hat{\sigma}_2 T_1 + \varphi_2 - \varphi_1,\quad \theta_2 = \hat{\sigma}_1 T_1 - \varphi_1 \qquad (24)$$

the averaging equations (20)-(23) becomes

$$\dot{a} = -\frac{\alpha_1}{2}a - \frac{\beta}{2\omega_1}b\sin\theta_1 - \frac{(\alpha_2-\gamma)\omega_2}{2\omega_1}b\cos\theta_1 - \frac{f_1}{2\omega_1}\cos\theta_2 \qquad (25)$$

$$a\dot{\varphi}_1 = \frac{\beta}{2\omega_1}b\cos\theta_1 - \frac{(\alpha_2-\gamma)\omega_2}{2\omega_1}b\sin\theta_1 - \frac{f_1}{2\omega_1}\sin\theta_2 \qquad (26)$$

$$\dot{b} = -\frac{\alpha_3}{2}a + \frac{\beta}{2\omega_2}a\sin\theta_1 - \frac{(\alpha_2+\gamma)\omega_1}{2\omega_2}a\cos\theta_1 \qquad (27)$$

$$b\dot{\varphi}_2 = \frac{\beta}{2\omega_2}a\cos\theta_1 + \frac{(\alpha_2+\gamma)\omega_1}{2\omega_2}a\sin\theta_1 \qquad (28)$$

Hence, the fixed points of Eqs. (25)-(28) are given by

$$\frac{\alpha_1}{2}a + \frac{\beta}{2\omega_1}b\sin\theta_1 + \frac{(\alpha_2-\gamma)\omega_2}{2\omega_1}b\cos\theta_1 + \frac{f_1}{2\omega_1}\cos\theta_2 = 0 \qquad (29)$$

$$a\,\sigma_1 - \frac{\beta}{2\omega_1}b\cos\theta_1 + \frac{(\alpha_2-\gamma)\omega_2}{2\omega_1}b\sin\theta_1 + \frac{f_1}{2\omega_1}\sin\theta_2 = 0 \qquad (30)$$

$$\frac{\alpha_3}{2}b - \frac{\beta}{2\omega_2}a\sin\theta_1 + \frac{(\alpha_2+\gamma)\omega_1}{2\omega_2}a\cos\theta_1 = 0 \qquad (31)$$

$$b\,(\sigma_1-\sigma_2) - \frac{\beta}{2\omega_2}a\cos\theta_1 - \frac{(\alpha_2+\gamma)\omega_1}{2\omega_2}a\sin\theta_1 = 0 \qquad (32)$$

There are three possibilities.

First case, $a = 0$, $b = 0$; this is trivial solution.

Second case, $a \neq 0$, $b = 0$; in this case the frequency response equation is given by

$$\sigma_1^2 a^2 + \frac{\alpha_1^2}{4}a^2 - \frac{f_1^{\,2}}{4\omega_1^2} = 0 \qquad (33)$$

Third case, $a \neq 0$, $b \neq 0$; this is the practical case, in this case the frequency response equations are given by

$$\sigma_1^2 a^2 + \frac{\alpha_1^2}{4}a^2 - \frac{f_1^{\,2}}{4\omega_1^2} - \frac{\beta^2}{4\omega_1^2}b^2 - \frac{(\alpha_2-\gamma)^2\omega_2^2}{4\omega_1^2}b^2 - \frac{(\alpha_2-\gamma)\omega_2 f_1}{4\omega_1^2}b = 0 \qquad (34)$$

$$(\sigma_1-\sigma_2)^2 b^2 + \frac{\alpha_3^2}{4}b^2 - \frac{\beta^2}{4\omega_2^2}a^2 - \frac{(\alpha_2+\gamma)^2\omega_1^2}{4\omega_2^2}a^2 = 0 \qquad (35)$$

4. Results and Discussions

The two-degree-of-freedom micro-electro-mechanical gyroscopes system (MEMS) under external excitations is studied. The solution of this system is determined up to the second order approximation using the multiple time scale perturbation. To study the behavior of the system of Eqs. (1)-(2), the Runge-Kutta of fourth order method was applied to determine the numerical solution of the given system. Fig. 2 illustrates the response for the non-resonant system where $\Omega_1 \neq \Omega_2 \neq \omega_1 \neq \omega_2$ at some values of the equation parameters . It is observed from this figure, the oscillation of the two modes of freedom micro-electro-mechanical gyroscopes system becomes stable and the steady state amplitudes x and y are about **0.0006** and **0.0008** respectively and the phase plane shows limit cycle, denoting that the system is free from chaos.

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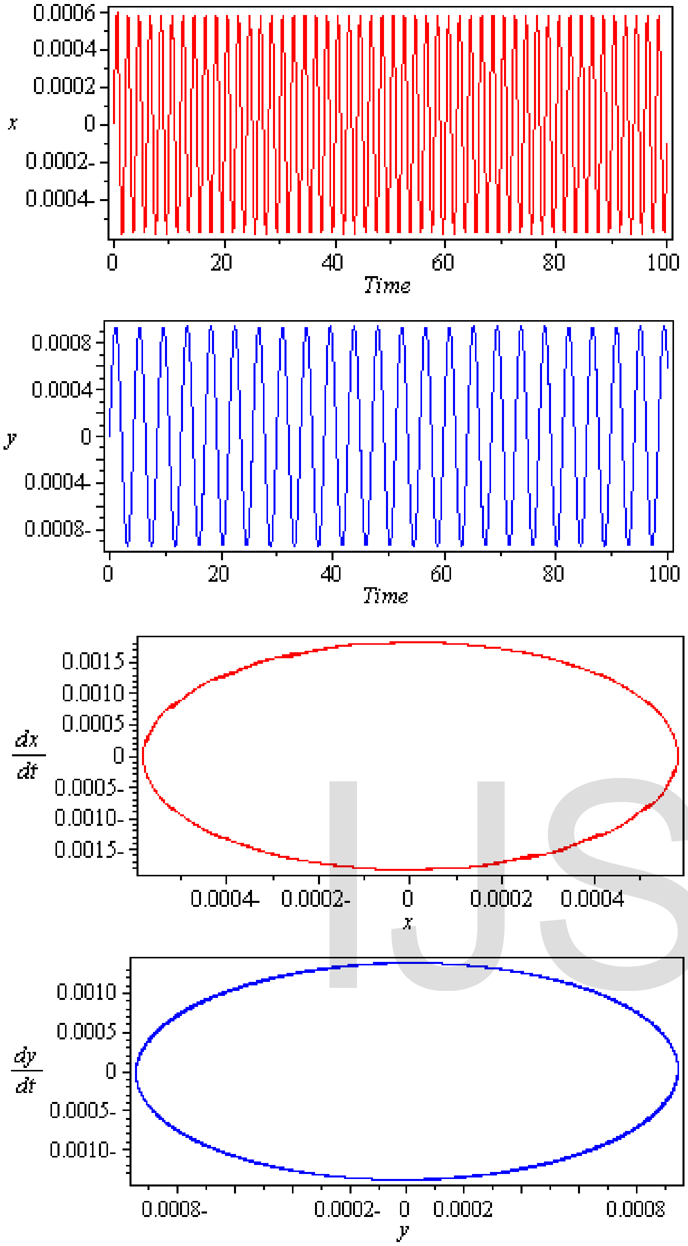

Fig. 2 Non-resonance system behavior (basic case)
α_1= 0.01, α_2= 0.002, α_3= 0.01, β= 0.1, γ= 0.04, f_1 = 0.2, f_2 = 0.5, ($\Omega_1 \neq \Omega_2 \neq \omega_1 \neq \omega_2$)

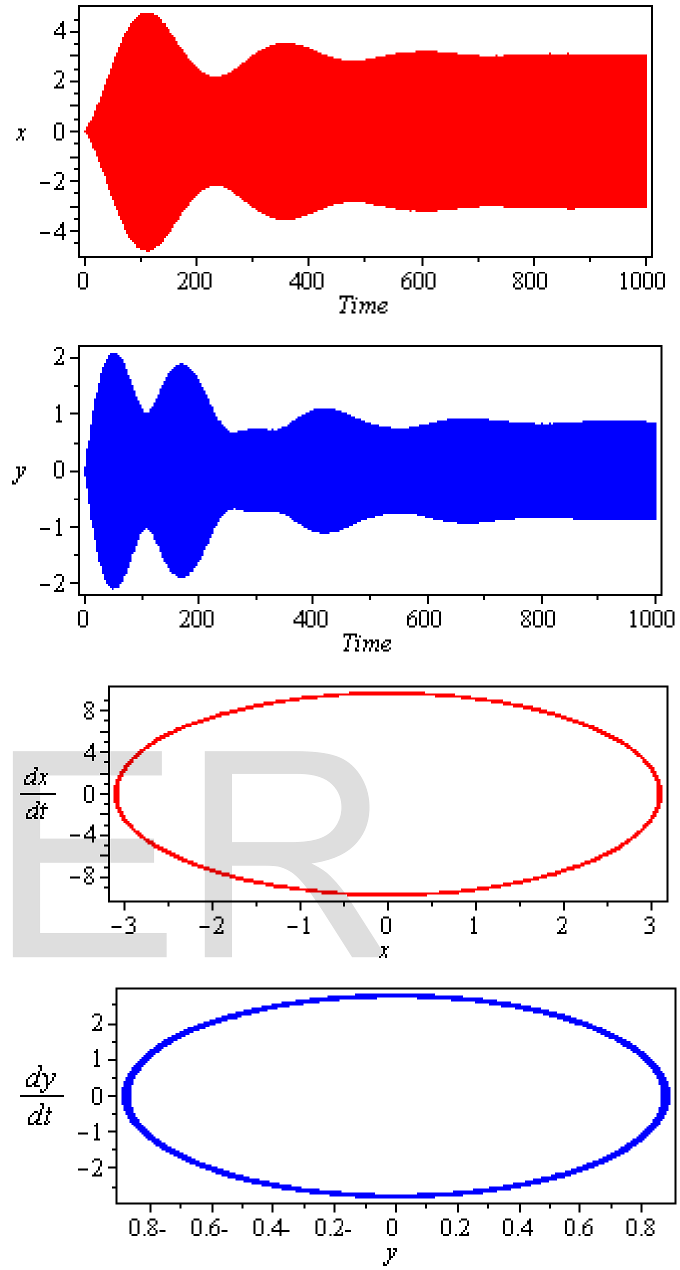

Fig. 3. Simultaneous primary and internal resonance case
α_1= 0.01, α_2= 0.002, α_3= 0.01, β= 0.1, γ= 0.04, f_1 = 0.2, f_2 = 0.5, ($\Omega_1 \cong \omega_1$, $\Omega_2 \cong \omega_2$, $\omega_1 \cong \omega_2$),

Fig. 3 shows that the time response of the simultaneous primaryand internal resonance case where ($\Omega_1 \cong \omega_1$, $\Omega_2 \cong \omega_2$, $\omega_1 \cong \omega_2$), which is one of the worst resonance cases. It is observed from this figure that the oscillation responses of the two modes of micro-electro-mechanical gyroscopes system start with increasing amplitude with tuned oscillations and the oscillations of the two modes becomes stable. From this figure we have that the amplitudes of the first and second modes system are increased to about **600%** and **180%** of the maximum amplitude f_2 respectively and the phase plane shows limit cycle.

4.1. Response curves and effects of different parameters

In this section, the steady state response of the given system at various parameters near the simultaneous primary and internal resonance case is investigated and studied. The frequency response equations given by Eqs. (34)-(35) are solved numerically at the same values of the parameters shown in Fig. 3.
Fig. 4a, show the steady state amplitudes of the first mode of

micro-electro-mechanical gyroscopes system against the detuning parameters σ_1.

Figs. 4 (b, f) show that the steady state amplitude of the first mode micro-electro-mechanical gyroscopes system is a monotonic decreasing function in the linear damping coefficient α_1 and the natural frequencies ω_1, ω_2. Figs. 4 (c, d, e, g) show that the steady state amplitude of the first mode micro-electro-mechanical gyroscopes system is a monotonic increasing function in the linear damping coefficient α_2 and the nonlinear parameters γ, β and the excitation force amplitude f_1.

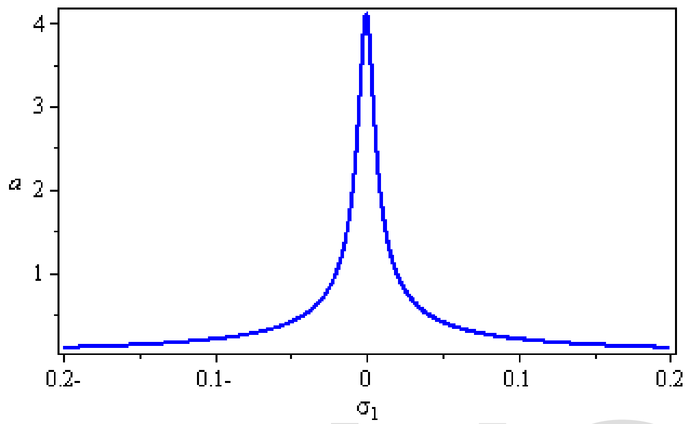

Fig.4a. Effects of the detuning parameter σ_1

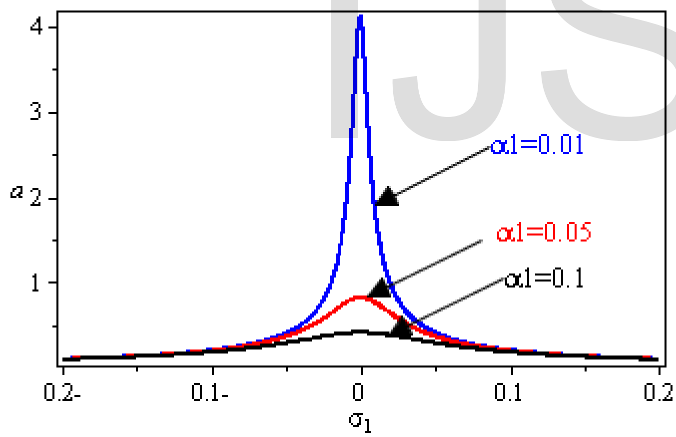

Fig.4b. Effects of the damping coefficient α_1

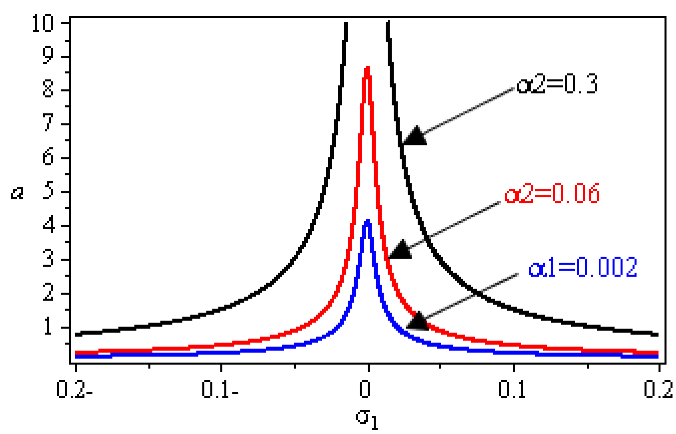

Fig.4c. Effects of damping coefficient α_2

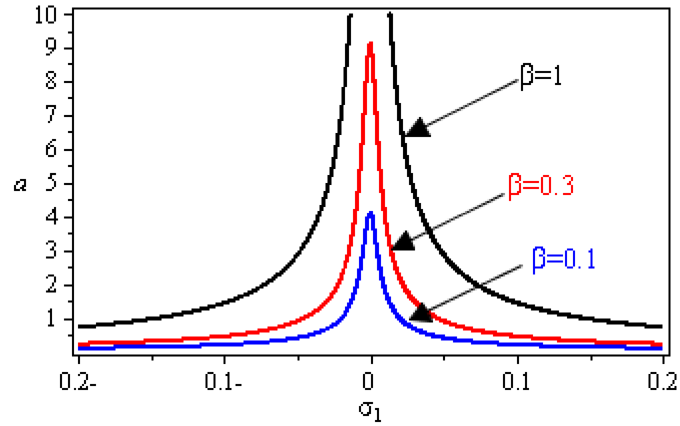

Fig.4d. Effects of the nonlinear parameter β

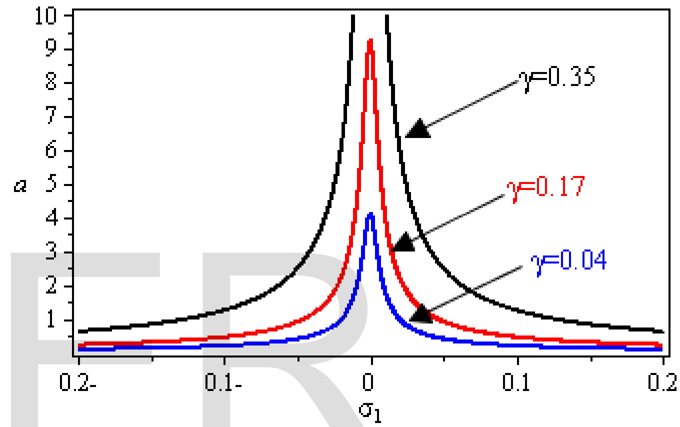

Fig.4e. Effects of the nonlinear parameter γ

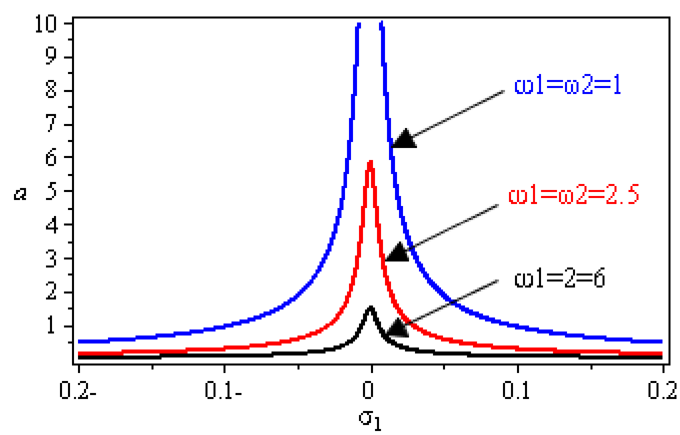

Fig.4f. Effects of the natural frequencies ω_1, ω_2

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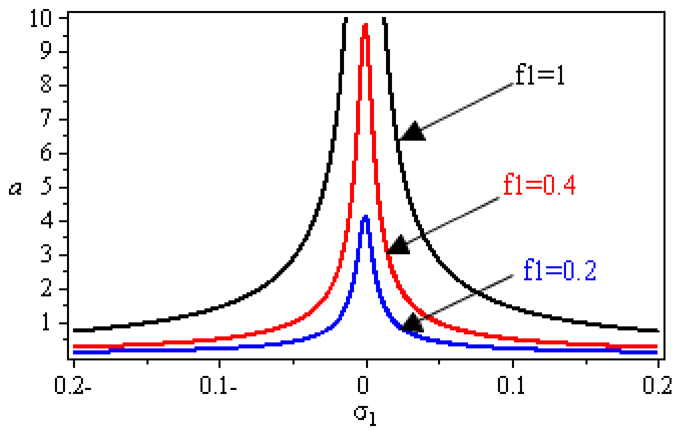

Fig.4g. Effects of the excitation amplitude f_1

Fig. 5a, show the steady state amplitudes of the second mode of mode micro-electro-mechanical gyroscopes system against the detuning parameters σ_2.

Figs. 5 (c, f) show that the steady state amplitude of the second mode micro-electro-mechanical gyroscopes system is a monotonic decreasing function in the linear damping coefficient α_3 and the natural frequencies ω_1, ω_2. Figs. 5 (b, d, e) show that the steady state amplitude of the second mode micro-electro-mechanical gyroscopes system is a monotonic increasing function in the linear damping coefficient α_2 and the nonlinear parameters γ and β.

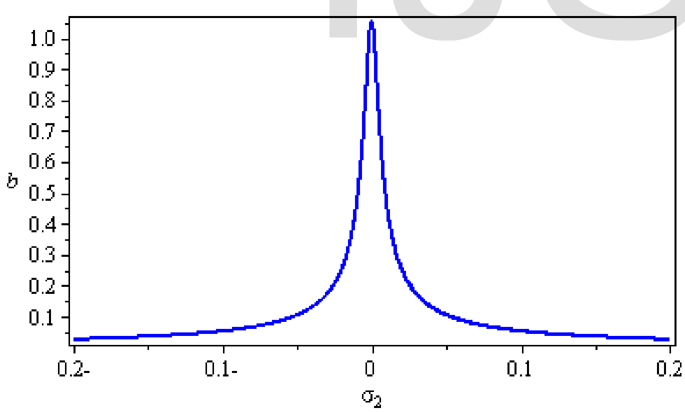

Fig.5a. Effects of the detuning parameter σ_2

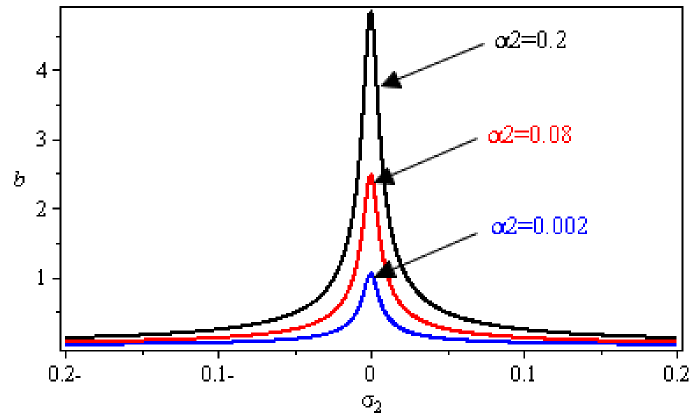

Fig.5b. Effects of damping coefficient α_2

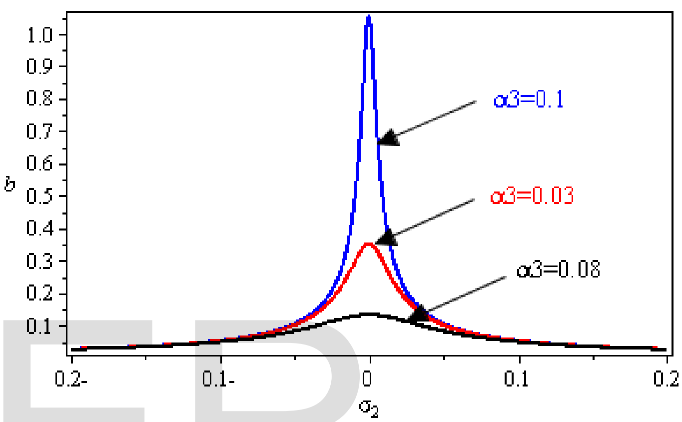

Fig.5c. Effects of damping coefficient α_3

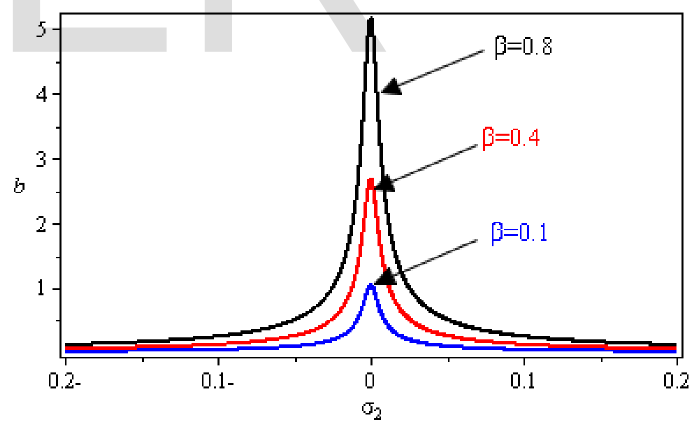

Fig.5d. Effects of the nonlinear parameter β

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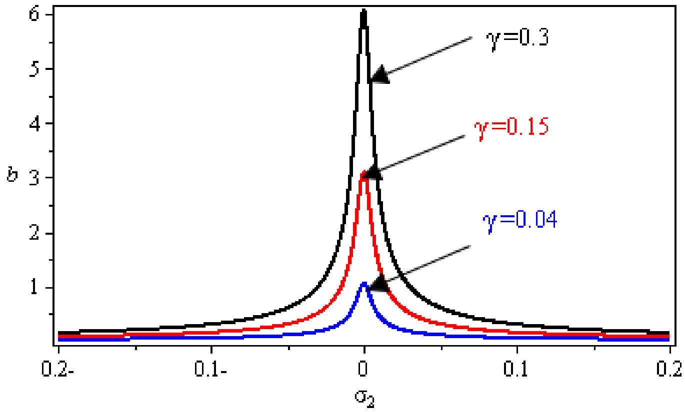

Fig.5e. Effects of the detuning parameter γ

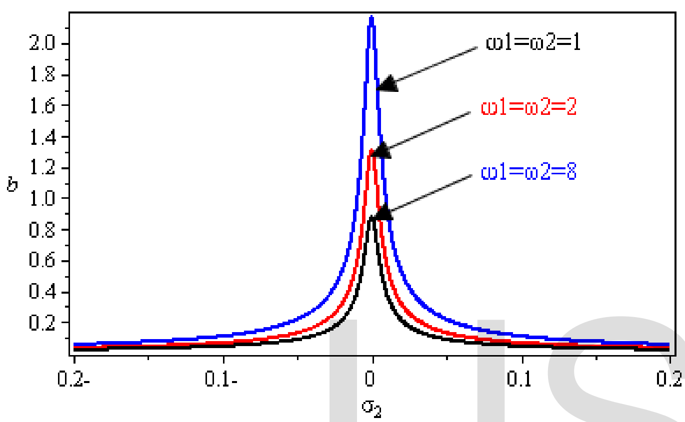

Fig.5f. Effects of the natural frequencies ω_1, ω_2

5. Comparison Study

In the previous work [14], studied the system of the micro-electro-mechanical gyroscopes system when subjected to external excitation forces. In our study, the response and stability of the system of two-degree-of freedom under to external excitation forces are investigated using the multiple time scale method. All possible resonance cases are extracted and investigated. The case of simultaneous principle primary resonance in the presence of 1:1 internal resonances is considered. The stability of the system is investigated using both frequency response equations and phase-plane method. It is quite clear that some of the simultaneous resonance cases are undesirable in the design of such system as they represent some of the worst behavior of the system.

5. Conclusions

The nonlinear responses of the micro-electro-mechanical gyroscopes system (MEMS) subjected to external excitations have been studied. The problem is described by a two-degree-of-freedom system of nonlinear ordinary differential equations. The case of simultaneous primary resonance in the presence of one-to-one internal resonance is studied by applying multiple time scale perturbation method. Both the frequency response equations and the phase-plane technique are applied to study the stability of the system. The effect of the different parameters of the system is studied numerically. From the above study the following may be concluded:

1- The simultaneous primaryand internal resonance case where ($\Omega_1 \cong \omega_1$, $\Omega_2 \cong \omega_2$, $\omega_1 \cong \omega_2$) is the worst cases and it should be avoided in design.
2- The amplitudes of the first and second modes system are increased to about 600% and 180% of the maximum amplitude f_2 respectively and the phase plane shows limit cycle.
3- The steady state amplitude of the micro-electro-mechanical gyroscopes system is a monotonic decreasing function in the linear damping coefficients α_1, α_3 and the natural frequencies ω_1, ω_2.
4- The steady state amplitude of the micro-electro-mechanical gyroscopes system is a monotonic increasing function in the linear damping coefficient α_2 and the nonlinear parameters γ, β and the excitation force amplitude f_1.

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